# The Crystal Structure of the $\boldsymbol{R}$ Phase, Mo-Co-Cr.* 

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#### Abstract

The crystal structure of the $R$ phase in the Mo-Co-Cr system, with atom ratio $30 \cdot 4-51 \cdot 3-18 \cdot 3$, has been determined by single-crystal X-ray diffraction analysis. The crystal has space group $C_{3 i}^{2}-R \overline{3}$, with 53 atoms per rhombohedral cell. The rhombohedral lattice constants are $a_{0}=9.011 \AA, \alpha=$ $74^{\circ} 27 \cdot 5^{\prime}$, hexagonal lattice constants $a_{0}=10 \cdot 903, c_{0}=19 \cdot 342 \AA$. The structure is closely related to those of other transition-group phases such as the $\sigma$ phase ( $\mathrm{Fe}-\mathrm{Cr}$ and many other systems), $P$ phase ( $\mathrm{Mo}-\mathrm{Ni}-\mathrm{Cr}$ and $\mathrm{Mo}-\mathrm{Ni}-\mathrm{Fe}$ ), $\chi$ phase ( $\alpha-\mathrm{Mn}, \mathrm{Mo}-\mathrm{Fe}-\mathrm{Cr}$ ), and $\mu$ phase (Mo-Co and other systems). The $R$ phase field lies directly between the $\sigma$ and $\mu$ phase fields on the $\mathrm{Mo}-\mathrm{Co}-\mathrm{Cr}$ ternary phase diagram. The $R$ phase exhibits 12 -fold (icosahedral), 14 -fold, 15 -fold, and 16 -fold atomic coordination, which are the coordinations shown in various combinations by the other phases mentioned. Like the others (except $\chi$ ), the $R$ phase has exclusively tetrahedral interstices. As with the $P$ and $\sigma$ phases, but to a somewhat lesser degree, interatomic distances in the $R$ phase agree with sums of characteristic radii, which are different for five- and six-coordinated ligands for each type of coordination. As in the case of the $P$ phase, the molybdenum content of the atomic sites increases with coordination from approximately zero for coordination 12 to approximately $100 \%$ for coordination 16.


## Introduction

The present investigation was carried out as a part of a program concerned with the investigation of the structures of some phases found in ternary systems of transition group elements by Beck and coworkers (Rideout et al., 1951). These and other phases which appear to be interrelated in their structural features and phase behavior include the $\sigma$ phase ( $\mathrm{Fe}-\mathrm{Cr}, \mathrm{Co}-\mathrm{Cr}$, and many other systems), the $P$ phase ( $\mathrm{Mo}-\mathrm{Ni}-\mathrm{Cr}$, $\mathrm{Mo}-\mathrm{Ni}-\mathrm{Fe}$ ), the delta phase ( $\mathrm{Mo}-\mathrm{Ni}$ ), the $\mu$ phase (Mo-Co and other systems), the $\chi$ phase $(\alpha-\mathrm{Mn}$, $\mathrm{Mo}-\mathrm{Fe}-\mathrm{Cr}$ ). (References for these structures are cited by Shoemaker, Shoemaker \& Wilson, 1957.) The $\beta$-tungsten and Laves-Friauf phases ( $\mathrm{MgCu}_{2}, \mathrm{MgZn}_{2}$, $\mathrm{MgNi}_{2}$ ) have structures that are less complex than the others cited, but related to them. Most of these phases appear to merit the designation of 'electron compound', since for a given structure the atomic composition is roughly determined by the electron content.

On the $\mathrm{Mo}-\mathrm{Co}-\mathrm{Cr}$ phase diagram at $1200{ }^{\circ} \mathrm{C}$., the $R$ phase, with atom ratio in the neighborhood of 30-49-21, lies between a $\sigma$ phase at about 0-32-68 to $20-48-32$ and a $\mu$ phase at about $36-49-14$ to 48-52-0. The $R$ phase, like the $P$ phase, was discovered by Beck and coworkers (Rideout et al., 1951).
Through the kindness of Prof. Paul A. Beck of the University of Illinois, specimens of $R$ phase, with atom

[^0]ratio $30 \cdot 4-51 \cdot 3-18 \cdot 3$ (by chemical analysis), annealed many hr . at $1200^{\circ} \mathrm{C}$., were made available to us for the X-ray crystallographic study here described. Some preliminary results have already been reported (Komura, Shoemaker \& Shoemaker, 1957).

## Unit cell and space group

In this work, as in the previous work on the $\sigma$ and $P$ phases, it was necessary to select many very small fragments from the pulverized alloy, mount them, and examine them by Laue photography in order to find and orient single crystals. A procedure frequently useful in the absence of morphological clues to symmetry and orientation is to take Laue photographs at a succession of trial azimuths until a mirror plane is encountered. In the present instance no mirror was found; with the weak patterns obtained (even after 24 hr . exposure) it was even difficult to determine whether fragments examined were single crystals, until by accident similar patterns were obtained with two different fragments, and later with two different settings of one fragment. The latter finding led to the discovery of a three-fold rotation axis. No Laue symmetry was found higher than $C_{3 i}-\overline{3}$, the only centrosymmetric class besides $C_{i}-\overline{1}$ that has no mirror plane. All later single crystal work was done with one fragment, a tiny flake less than 0.05 mm . in its longest dimension, mounted on a glass fiber with fused shellac.

The lattice was established as rhombohedral with the aid of a gnomonic projection of a Laue photograph. With Weissenberg and oscillation photographs the axial lengths were obtained, although very few re-

Table 1. Powder photographic data*

| hkl | $\frac{I}{d_{c a l}}$ | $I_{\text {cal }}$ | $\frac{1}{d_{o b s}}$ | $I_{o b s}^{\dagger}$ | hk $\ell$ | $\frac{1}{d_{\text {cal }}}$ | $I_{\text {cal }}$ | $\frac{1}{d_{o b s}}$ | $I_{\text {obs }}$ | hat | $\frac{1}{d_{\text {cal }}}$ | $I_{\text {cal }}$ | $\frac{1}{\mathrm{~d}_{\text {obs }}}$ | $I_{\text {obs }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 131 | . 3854 | 3.1 | . 3847 | $\pm$ | 333 | . 5718 | 2.5 |  |  | 710 | . 7996 | 22.7 | . 7998 | Vr |
| 312 | . 3956 | 4.3 |  |  | 416 | . 5760 | 8.6 | . 5764 | $f(\mathrm{~b})$ |  |  |  | . 8029 | $f($ ) |
| 223 | . 3983 | 6.22 |  |  | 0.1 .11 | . 5784 | 0.5 | . 5800 | $\pm$ | 265 | . 8063 | 12.6 | . 8074 | $\pm$ |
| 027 | . 4193 | 1.8 | . 4194 | vf |  |  | ** |  |  | 618 | . 8083 | $1.8\}$ | . 8098 |  |
|  |  |  | . 4217 | 1 | 3.0 .12 | . 6970 | 2.0 | . 6985 | $f(b)$ | 529 | . 8087 | $29.1\}$ | . 8098 | ms |
| 401 | . 4268 | 1.6 ) | . 4275 | ww | 612 | . 7022 | 8.0 | . 7031 | $f(b)$ | 713 | . 8145 | 23.9 \} | . 8162 |  |
| 018 | . 4269 | $18.4\}$ |  |  | 0.2.13 | . 7046 | 1.8 |  |  | 3.2 .13 | . 8153 | 10.5 | .8162 | s |
| 134 | . 4344 | 11.0 | . 4350 | $f$ | 0.2.13 | . 7071 | 4.4 |  |  | 3.1 .14 | .8183 | $6.7\}$ | . 8189 | vf |
| 042 | . 4361 | 1.8 |  |  | 0.4 .11 | . 7091 | 3.2 |  |  | 1.5 .11 | . 8192 | 1.4 | . |  |
| 306 | . 4440 | 6.8 | . 4463 | vf | 0.4.11 | . 7091 | 3.2 |  |  |  |  |  | .8217 | f |
| 217 | . 4577 | 78.4 | . 4581 | vs | 158 | . 7203 | 0.9 | .7157 | w(b) | ${ }^{707}$ | .8250 | 12.1 1 | . 8264 | W |
| 315 | . 4611 | 84.0 | . 4614 | vs | 1339 | . .7207 | $\left.\begin{array}{l}0.9 \\ 4.2\end{array}\right\}$ |  |  |  |  |  |  |  |
| 321 | . 4646 | . 5 5 | . 4670 | vw | 526 | . 7306 | 9.5 |  |  |  | $\begin{aligned} & .8288 \\ & .8293 \end{aligned}$ | $\left.\begin{array}{l}8.0 \\ 5.4\end{array}\right\}$ | . 8313 | $\mathbf{f}$ |
| 208 | . 4647 | 4.25 | . 4670 | w | 0.1 .14 | . 7314 | 5.94 | . 7318 | * |  | . 8337 | 1.52 |  |  |
| 404 | . 4714 | 9.1 |  |  | 2.3.11 | . 7325 | 2.4 |  |  | $1.0 .16$ | . 8339 | 2.15 |  |  |
| 232 | . 4731 | 70.5 | . 4734 | vs | 0.5 .10 | .7401 | 0.9 |  |  | 3.0.15 | . 8380 |  | . 8376 | mv |
| 226 | . 4804 | 1.9 |  |  | 701 | . 7432 | 5.2 | .7432 | $f(\mathrm{~b})$ | 630 | . 8407 | 5.3 | . 8408 |  |
| 410 | . 4854 | 62.2 | . 4857 | vs | 443 | . 7500 | 1.5 | .7499 | $f$ | 627 | . 8452 | 1.0 | . 8450 | vf |
| 045 | . 4963 | 46.4 | . 4960 | $s$ | 2.0 .14 | . 7541 | 0.6 | . 7510 | $f(\mathrm{~b})$ ? | 081 | . 8489 | 5.2 | . 8498 | vor |
| 128 | . 4996 | 51.4 , | . 5008 | vs | 2.4 .10 | . 7625 | 0.6 | . 7587 | $f(\mathrm{~b})$ ? | *hkl: based on hexagonal axes <br> $1 / \alpha_{\text {cal }}$ : from final lattice constants |  |  |  |  |
| 119 | . 5001 | 33.6 \} | . 5008 | vs |  |  |  | . 7625 | $f(\mathrm{~b})$ ? |  |  |  |  |  |
| 324 | . 5059 | 36.9 | . 5056 | m | 348 | . 7656 | 1.0 | . 7667 | $f$ |  |  |  |  |  |
| 413 | . 5095 | 28.8 | . 5080 | W | 704 | . 7697 | 8.77 | . 7698 | TV | $I_{\text {cal }}=\mathrm{KmF}^{2}$ : from single-crystal in- |  |  |  |  |
| 137 | . 5261 | 8.8 | . 5217 | マ | 262 | . 7707 | $2.3)$ | .769 | W | $1 / \alpha_{\text {obs }}$ : average of $\mathrm{Fe} K \alpha$ and CrK $\alpha$ data |  |  |  |  |
| 1.0 .10 | . 5277 | $15.1\}$ | . 5289 | min | 0.0 .15 | . 7754 | 6.3 | .7726 | $f$ ? | Iobs: average powder-line intensity |  |  |  |  |
| 235 | . 5291 | $12.6\}$ | -5209 | mex | 1.2 .14 | . 7761 | 17.5 | . 7780 | nsw | ** No significant lines predicted, and none observed, in this range |  |  |  |  |
| 051 | . 5321 | 2.5 |  |  | 5.1 .10 | .7842 | 1.4 | . 7848 |  |  |  |  |  |  |
| 502 | . 5396 | 9.7 | . 5382 | vr | 535 | . 7852 | $20.3)$ |  |  | $\dagger_{\mathrm{s}}=$ strong, $\mathrm{m}=$ medium, $\mathrm{w}=$ weak, $f=$ faint, $b=$ broad, $v=$ very |  |  |  |  |
| 0.2 .10 | . 5587 | 0.6 | . 5575 | VW | $609$ <br> 4.1 .12 | $.7876$ | $\left.\begin{array}{l} 15.4 \\ 16.5 \end{array}\right\}$ | . 7876 | ${ }^{\text {mis }}$ |  |  |  |  |  |
| $\begin{aligned} & 241 \\ & 309 \end{aligned}$ | $\begin{aligned} & .5628 \\ & .5634 \end{aligned}$ | $\left.\begin{array}{r} 2.3 \\ 16.2\} \end{array}\right\}$ | . 5624 | $v$ (b) | 6.1 .12 | . 7913 | 1.3 1.3 | . 7987 | * |  |  |  |  |  |
| 054 | . 5685 | 2.1 | .5690 | W | $\begin{aligned} & 1.1 .15 \\ & 4.2 .11 \end{aligned}$ | $\begin{aligned} & .7968 \\ & .7984 \end{aligned}$ | $\left.\begin{array}{l} 1.3 \\ 3.8 \end{array}\right\}$ | . 7972 | f |  |  |  |  |  |

flections were observed on the equatorial Weissenberg photograph.

The lattice constants were eventually refined with powder data obtained with $\mathrm{Fe} K \alpha$ and $\mathrm{Cr} K \alpha$ radiation. The powder lines, measured from photographs taken on a 114.59 mm . Philips powder camera (Straumanis arrangement), were indexed with the aid of a predicted powder diagram based on three-dimensional single-crystal intensity data, and the lattice constants were refined by least squares by the procedure used for the $P$ phase (Shoemaker, Shoemaker \& Wilson, 1957). The powder data are given in Table l. The final values of the lattice constants are:

$$
\begin{array}{cc}
\text { Rhombohedral } & \text { Hexagonal } \\
a_{0}=9.011 \pm 0.005 \AA & a_{0}=10.903 \pm 0.005 \AA \\
\alpha=74^{\circ} 27.5^{\prime}+2^{\prime} & c_{0}=19.342 \pm 0.011 \AA
\end{array}
$$

The density was measured by displacement in a pyenometer. Six independent measurements yielded an average value of $9.011 \mathrm{~g} . \mathrm{cm} .^{-3}$ with an average deviation of 0.013 . With the given composition this corresponds to 50.92 atoms per rhombohedral cell, suggesting the number 51 . Our experience with the
$P$ phase (which apparently had a density lower than the calculated value because of slag inclusions) suggested that the number of atoms could be higher52,53 , or even possibly 54.
The only two space groups compatible with the rhombohedral lattice and the observed Laue symmetry are $R 3$ and $R \overline{3}$. The latter was provisionally assumed in the subsequent work, and eventually found to be the correct one.

## Determination of the structure

The provisionally assumed space group $R \overline{3}$ has two sets of 3 -fold special positions, but these did not appear to be occupied since peaks required by them in the basal plane Patterson projection (Fig. 1) are not observed. The only other special positions lie on the three-fold rotation axes. The general positions in this space group are 6 -fold. With 51 to 53 atoms per rhombohedral cell (with a $19 \cdot 3 \AA$ hexagonal c-repeat) it is clear that three to five must lie on the three-fold axis. With 54 atoms, the number could be either zero or six. However, if there are no atoms on the axis, or indeed any fewer than four per cell, it is clear that
interstices must be present on the axis that are not tetrahedral. Although a few models with one or a few octahedral interstices were examined, the virtual absence of other than tetrahedral interstices in the other known transition alloy complex structures made it seem probable that only tetrahedral interstices are present in the $R$ phase. Thus it appeared that there are four, five, or perhaps six atoms on the axis per unit cell.


Fig. 1. Patterson projection on basal plane.
This circumstance requires that four to six atoms be superimposed in projection on the basal plane, making it probable that in such a projection they behave like a single 'heavy atom', making the signs positive for most or all of the large structure factors in the ( hk 0$)_{\text {hex }}$ zone. The basal plane Patterson projection (Fig. I) should therefore be expected to resemble closely an electron-density projection of the structure. In this projection a cluster of peaks surround the origin and other lattice points. The strong peaks in each cluster suggest immediately a projection of a regular icosahedron.

These considerations led us to consider possible configurations of icosahedra, and other of the coordination polyhedra that were encountered in the $\sigma$ and $P$ phases, with their four, five or six central atoms on the three-fold axis, and with sharing of vertices or faces.

After a number of trials, a promising arrangement was found with five atoms. This consisted of three icosahedra and two CN 16 polyhedra (octicosahedra), the icosahedra sharing triangular faces with each other and with the CN 16 polyhedra, and the latter mutually sharing a six-membered ring. The atoms of the CN 16 polyhedra, except for those in the six-membered ring, project together with all but one kind of icosahedral atoms ( $B 2$ ) onto the strong peaks of the Patterson projection. The weak peaks are reasonably well accounted for by the remaining atoms (6-ring and $B 2$ ).

The chain of polyhedra accounts for one one-fold and two two-fold sets of special positions, and eight six-fold sets of general positions, totalling 53 atoms


Fig. 2. $R$ phase structure: chain of packing polyhedra along three-fold axis.
per rhombohedral cell. The structure consists of such chains packed together, one running along each threefold axis. Two configurations for the chain, related by a reflection in the basal plane or a $60^{\circ}$ rotation, had to be considered; the construction of models of both showed clearly that one and only one of them yields a plausible structure from the packing standpoint. In this one, it was possible to identify the coordinations of all of the atoms in terms of the CN 12, 14,15 , and 16 polyhedra previously encountered in the $\sigma$ and $P$ phases. Moreover, this structure yielded three-dimensional calculated structure factors in reasonable agreement with the observed. The chain is shown in its correct configuration, in relation to the rhombohedral unit cell, in Fig. 2.

## Parameter refinement

Intensity data for refinement were obtained by visual estimation of intensities of diffraction spots on mul-tiple-film equi-inclination Weissenberg photographs, with the aid of intensity strips, in the usual way. The crystal was rotated around the three-fold axis for layers $l=0$ to $l=16$, and around $[110]_{\text {hex. }}$ for the first five layers. Copper $K \alpha$ radiation was used. Owing in part to the smallness of the crystal (and despite exposures of $150-200 \mathrm{hr}$.), but also in large part to the rather unusual distribution of intensities that is peculiar to structures of this kind, 521 of the 900 nonequivalent planes accessible to observation were too weak to be observed.

Because at the time the approximate structure was obtained our computing facilities for least-squares refinement were not yet ready, an interim attempt was made to obtain some 'refinement' without recourse to intensity data, by attempting to minimize devia-
tions between interatomic distances calculated from atomic coordinates and distances obtained from radius sums for the corresponding polyhedra (as found for the $P$ phase). This was done by a procedure tantamount to a diagonal least-squares treatment; in each cycle, the distances were calculated, and the differences between them and radius sums were averaged to obtain shifts. Six cycles were carried through to approximate convergence, accomplishing about half of the ultimate refinement; the average deviation of the parameters from their final values was reduced from 0.0084 to 0.0039 . These calculations were of value in showing that the structural geometry itself imposes certain limits on the degree of ultimate agreement between actual distances and radius sums, since a number of large ( $0 \cdot 1-0 \cdot 2 \AA$ ) deviations remained after this treatment; these were, for the most part, for the same atom pairs for which large distance deviations were found after ultimate refinement with X-ray intensities.

Intensity refinement was carried out by the method of least squares on the IBM 704 computer, with the program NY XR 1 developed by D. Sayre and modified by V. Vand. This program omits off-diagonal normal-equation coefficients. All observations included in the refinement were weighted equally. In the early stages, all planes accessible to observation were included, but in the later stages the unobserved planes were omitted. Also omitted in the later stages were twenty-three reflections which seemed liable to suspicion of abnormal error or mis-identification. All of these were observed considerably stronger than calculated, and most were near the edge of the sphere of reflection. As the films contain many random spots clearly due to misoriented attached crystal fragments, it is likely that some such spots were mistaken for spots due to the main crystal.

To determine approximately the occupancy of atomic sites with molybdenum atoms, some refinement with respect to 'atomic numbers' or 'atomic scale parameters' for the several kinds of atomic sites was carried out. This was done with a modification of NY XR 1, in which these parameters replaced the temperature factor parameters.

Following thirteen cycles of preliminary refinement, and two cycles of atomic number refinement, the form factors for the various sites were adjusted in accord with the atomic numbers obtained and three additional cycles of positional and temperature factor refinement and one of temperature factor refinement alone were carried out. In these final cycles, Thomas-Fermi-Dirac (TFD) form factors (Thomas \& Umeda, 1957) were used, in linear combinations of cobalt and molybdenum suggested by the atomic number results. These form factors were corrected for anomalous dispersion in accordance with the table of Dauben \& Templeton (1955). The final values of the positional, temperature factor, and atomic number parameters are given in Table 2, together with the approximate molybdenum percentages assumed for calculating structure factors. The positional parameters are given in terms of the hexagonal cell ('obverse' relationship; rhombohedral lattice points at $\left.0,0,0 ; \frac{2}{3}, \frac{1}{3}, \frac{1}{3} ; \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$. In Table 3 the final calculated structure factors are compared with the observed. The structure factors are scaled to the 159 -atom hexagonal cell. The final reliability factor, $R$, was $9 \cdot 6 \%$ for the reflections included in the refinement.

## Discussion

The structure, shown in various aspects in Figs. 2 to 6 , is of lower symmetry than any of the other transition group alloy phases of known structure that have been here mentioned; this and its evident complexity make detailed comparisons with the other structures difficult. Some direct structural relationships with the $\sigma$ phase (Bergman \& Shoemaker, 1954) and $P$ phase (Shoemaker, Shoemaker \& Wilson, 1957) can be seen, however, by means of an axis transformation,

$$
\begin{aligned}
\left(\begin{array}{l}
\mathbf{a}^{\prime} \\
\mathbf{b}^{\prime} \\
\mathbf{c}^{\prime}
\end{array}\right) & =\left(\begin{array}{rrr}
\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\
-\frac{5}{3} & \frac{5}{3} & \frac{2}{3} \\
-\frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{array}\right) \cdot\left(\begin{array}{l}
\mathbf{a}_{\text {nex. }} \\
\mathbf{b}_{\mathrm{bex} .} \\
\mathbf{c}_{\mathrm{hex} .}
\end{array}\right) \\
& =\left(\begin{array}{rrr}
1 & 0 & 0 \\
-1 & 4 & -1 \\
0 & 1 & 0
\end{array}\right) \cdot\left(\begin{array}{l}
\mathbf{a}_{\mathrm{rh}} \\
\mathbf{b}_{\mathrm{rh}} \\
\mathbf{c}_{\mathrm{rh}}
\end{array}\right),
\end{aligned}
$$

Table 2. Atomic parameters

| Atom | Pos. | CN | $x$ | $y$ | $z$ | $B$ | $Z_{0}$ | \% Mo* | $Z_{\text {assum. }} \dagger$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A1 | $3(b)$ | 12 | 0 | 0 | $\frac{1}{2}$ | 1.72 | 27.3 | 11 | 26.8 |
| A2 | 6 (c) | 12 | 0 | 0 | $0 \cdot 3044$ | $1 \cdot 17$ | $25 \cdot 3$ | 0 | $25 \cdot 0$ |
| A3 | 18(f) | 12 | $0 \cdot 1596$ | $0 \cdot 2470$ | 0.0020 | 0.59 | $25 \cdot 0$ | 0 | $25 \cdot 0$ |
| A4 | 18(f) | 12 | 0.0509 | $0 \cdot 2790$ | $0 \cdot 1000$ | $0 \cdot 43$ | $25 \cdot 1$ | 0 | $25 \cdot 0$ |
| A5 | 18(f) | 12 | 0.0212 | $0 \cdot 1393$ | $0 \cdot 1962$ | 1.28 | 26.3 | 11 | 26.8 |
| A6 | 18(f) | 12 | 0.2250 | 0.1969 | 0.2685 | $0 \cdot 40$ | $25 \cdot 3$ | 0 | $25 \cdot 0$ |
| B1 | 18(f) | 14 | $0 \cdot 1759$ | $0 \cdot 1265$ | 0.3969 | 0.59 | $35 \cdot 2$ | 62 | $35 \cdot 2$ |
| $B 2$ | 18(f) | 14 | $0 \cdot 1132$ | 0.2687 | $0 \cdot 4652$ | 0.76 | $33 \cdot 8$ | 53 | $33 \cdot 8$ |
| $C 1$ | 18(f) | 15 | 0.0330 | 0.2579 | 0.3183 | 0.59 | $37 \cdot 4$ | 76 | 37.5 |
| D1 | 6 (c) | 16 | 0 | 0 | 0.0735 | 0.38 | $44 \cdot 4$ | 100 | 41.5 |
| D2 | 18(f) | 16 | 0.2671 | 0.2218 | $0 \cdot 1222$ | $0 \cdot 44$ | 41.8 | 100 | 41.5 |

[^1]$\dagger$ Effective form factor at $\theta=0$, based on $\%$ Mo assumed; 0.5 and 2.0 electrons subtracted from $f_{M o}$ and $f_{\text {Co }}$ respectively.

Table 3. Calculated and observed structure factors for $R$ phase


Table 3 （cont．）

|  | 10 K 9 | 4 | 196－202 ${ }_{\text {77 }}$ | -8 -5 | $\begin{array}{ll} 302 & 310 \\ 146 & 153 \mathrm{~m} \end{array}$ |  | $6 \times 14$ | 5 | 19－128 M | 2 | 642－670 |  | $1 \times 23$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| －${ }_{5}^{8}$ | 124 98 95 |  |  | － 2 | 108－ 133 H | $=3$ | ${ }_{536} 1513 \times$ |  | －$\times 16$ | 5 | ${ }_{18}^{60}$－${ }_{144} 14$ |  | $1 \times 23$ |
| － 2 | 95－137 ${ }^{\text {95 }}$ |  | 4 K 11 |  | 11 K 12 | －${ }_{1}$ | ${ }_{152}^{151}$－ 154 m | － 3 | 69119 |  | 1 K 19 | 2 | 301－310 |
| 1 | 27.91 M | － 1 | 181－172 |  | $11 \times 12$ | 4 | $109{ }^{117}{ }^{\text {m }}$ | 0 | 645618 |  |  |  | 2 K 23 |
|  | 11 K 9 | ${ }_{5}^{2}$ | $5{ }^{5}$ | － 7 | $\begin{array}{ll}170 \\ 420 & 234 \\ 428\end{array}$ |  | $7 \times 14$ | 6 | 63－${ }^{63} 88 \mathrm{M}$ | $\bigcirc$ |  |  |  |
|  |  | ${ }_{8}$ | 69 7\％M |  |  |  |  |  |  | 6 | 499－397 | 3 | $167^{-152}$ |
| $-10$ | S930－915 ${ }^{39}$ |  |  |  | 12 K 12 | － | $188-150 \mathrm{M}$ |  | 5 K 16 |  |  |  |  |
| 27 | ${ }_{70} 990{ }^{575}$ \％ |  | 5 K 11 | －6 | 26 － 61 M | － 1 | 70 780 M |  |  |  | 2 K 19 |  | $3 \times 23$ |
| －1 | 14 91m | －3 | 543  <br> 55 507 <br> 135  |  |  |  |  | $\begin{aligned} & 2 \\ & 1 \end{aligned}$ | 12－135 M | 1 | 171 <br> $120-185$ <br> 137 | 1 | 245224 |
|  | 12 K 9 | 3 |  |  | OK 13 |  | $8 \times 14$ |  | 268289 | 4 | 120－137 M |  |  |
|  |  | 6 | 135191 |  |  | － 6 | 28－146 M |  | $6 \times 16$ |  | 3 k 19 |  | $0 \times 24$ |
| －9 | 123 151 <br> 932  <br> 16  |  | 6 K 11 | 2 | ${ }_{108}^{321}-{ }^{278}$（39 m | －3 | 374 37 |  | 207208 |  | 414－467 | 0 | 152－93 M |
| － 3 | 653－596 |  | 6 K 11 | 8 | ${ }_{45}{ }^{139}$－ 139 | 0 | 112－67M | －1 | 207 14 | $\frac{1}{2}$ | 105 ＝ 251 | 0 | 152－93 $\quad$－ |
|  |  | － 5 | 26－120－ 139 |  | 1 k 13 |  | 9K14 | 2 | 33 124 M |  |  |  | $1 \times 24$ |
|  | $0 \times 10$ | 1 | ${ }_{102} 152 \mathrm{M}$ |  | k |  | K 14 |  | $7 \times 16$ |  | $4 \times 19$ | 1 | 61 － 34 M |
|  |  |  | 517622 | 0 | 119 59 M | －8 | 94－123 M |  |  | 0 | 160157 M |  |  |
| 5 | ${ }_{220}^{157-124}$ |  | 7 K 11 | 6 |  | －${ }^{-1}$ | 7 2 | －${ }^{6}$ | 32 12 |  | $5 \times 19$ |  |  |
| 8 | 308344 |  |  | 9 | 82 － 100 M | 1 | 334316 | 0 | 101－126M |  | S ${ }^{19}$ |  | 66.81 M |
|  | $1 \times 10$ | 4 | 78－148 ${ }^{\text {m }}$ |  | 2 K 13 |  | 10 l | 3 | 153157 | 1 | 119－141 M |  |  |
|  |  | 2 |  |  | 2 K 13 |  | 10 K 14 |  | $8 \times 16$ |  | $6 \times 19$ |  |  |
| 0 | 858734 | 5 | 1584 M | 1 | 244－209 | $-7$ | 708 |  |  |  |  |  |  |
| 5 | 193 $190{ }^{6}$ |  | $8 \times 11$ | 4 | $129-135 \%$ | －1 |  | －${ }^{5}$ | 680 139 | 2 | 44147 |  | notes |
| 9 | 134－117 M |  |  |  |  |  |  | － | 19－95 |  | 7 K 19 |  |  |
|  | 2 k 10 | －8 | ${ }_{282}^{170}$－ 203 |  | 3 K 13 |  | 11 K 14 |  |  |  |  |  |  |
|  |  |  | 11.150 M | － 1 | 270236 | －6 | 563568 |  | 9 K 16 | 0 | 32188 |  | Hon．VALUE |
| 1 | 186－220 | 3 | 3109 M | 2 | 14391473 | －3 | 577426 | － 7 | 62－105 M |  |  |  | E IS ESTIMATE |
| 4 | 98  <br> 715 163 <br> 16  |  | 9 K 11 | ${ }_{8}^{5}$ | ${ }_{283}^{125}$－${ }_{289}^{150} \times$ |  |  | － 1 | ${ }_{762} 322-295$ |  | $0 \times 20$ |  | ImIMUM OBSERV－ |
|  |  |  |  | 8 |  |  | OK15 |  |  | 1 | $124 \quad 305$ |  |  |
|  | $3 \times 10$ | －8 | 318 |  | $4 \times 13$ |  |  |  | $10 \times 16$ | 4 | $482-473$ |  |  |
| －1 | 74.82 m | － 3 | ${ }_{67} 101-133 \mathrm{M}$ | 3 | $40-117 \mathrm{M}$ | ${ }_{3}^{0}$ | 1307 304 | －6 | 64.88 M | 7 | $51 \quad 64$ |  | OMITED FROM |
|  | 237－221 | 1 | 140－119 m | 0 | 106123 M | 6 | 79 141m | －3 | $53 \quad 77 \mathrm{M}$ |  | $1 \times 20$ |  | on of amormal |
| 5 | 53－ 1140 m |  |  | 3 | $4{ }^{4} 3148 \mathrm{M}$ | 9 | 2－93M |  |  |  |  |  | OR MISIDEN－ |
| 8 | 41 － 110 m |  | 10 K 11 | 6 | 742920 |  | $1 \times 15$ |  | OK 17 | 2 | $\begin{array}{rl} 50 & 247 \\ 0 & 119 \mathrm{M} \end{array}$ |  | cation． |
|  | 4 $\times 10$ | 7 | 56－135 M |  | K 13 |  |  |  |  |  |  |  |  |
| －3 |  | －1 | $\begin{array}{ll}267 & 310 \\ 410\end{array}$ | － 2 |  | 4 |  | 4 | ${ }_{116}^{11-1770 M}$ |  | 2 K 20 |  |  |
| 0 | $7^{73}{ }^{107} 107$ | －1 |  | 1 | ${ }^{179}$－ $144 \mu$ | 7 | 289 309 | 7 | ${ }_{63}^{116} 1172 \mathrm{M}$ | 0 | ${ }^{58}$－ 157 M |  |  |
| ${ }_{6}$ | 663 <br> 15 |  | 11 K 11 | 4 | 9141 M |  |  |  |  | 3 | 172－135 M |  |  |
|  |  |  | 325－329 |  | 6 K |  | 2 K 15 |  | 1 K 17 |  | 3 $\times 20$ |  |  |
|  | 5 K 10 | － 6 $=-3$ | ${ }_{41}^{22}=119 \mathrm{M}$ |  | 44． 141 M | 1 | $20.18{ }^{34} \times$ | 2 | $100-293$ |  |  |  |  |
|  |  |  | 41 － 109 M | － 4 | 44＊ $14{ }^{\text {\％M }}$ | 2 | 193－119 M | 5 | 336－353 | 1 | 112－153 |  |  |
| 1 | 188 ＝ 1130 m |  | 12 k 11 | $-\frac{1}{2}$ | $225-{ }^{27}{ }^{144}{ }^{\text {m }}$ | 5 | $\begin{array}{ll}183 & 141 \\ 188\end{array}$ | 8 |  |  | K 20 |  |  |
| 4 |  | － 6 | 139－137 | 5 | 22－102M |  | $3 \times 15$ |  | $2 \times 17$ | 2 | 111123 |  |  |
|  | $6 \times 10$ | － 5 | 427－387 |  | $7 \times 13$ |  | 391－368 | ${ }_{3}^{0}$ |  |  | $5 \times 20$ |  |  |
|  |  |  |  | － 6 | 641866 | 3 | 54 137 m | 3 | 36－124 M |  |  |  |  |
| 4 | 101－126 M |  | $0 \times 12$ | －3 | 620611 | 6 | 167123 M |  |  | 0 | 88－128 M |  |  |
| $\frac{1}{2}$ | ${ }_{113}^{300}-{ }_{144} \mathbf{2 6 4}$ | 0 | 233－196 | 0 | ${ }_{30}^{23}-148{ }^{14}$ |  | 4 K 15 |  | K |  | 6 K 20 |  |  |
| 5 | 109－119 M | 3 | 155 114 M |  |  |  |  | 1 | 333－354 |  |  |  |  |
|  | 7 K 10 | 9 | $\begin{aligned} & 40=166 \text { M } \\ & 259 \end{aligned}$ |  | $8 \times 13$ | － 2 | 108－117 M |  |  | 2 | 3 － 93 M |  |  |
|  | 7 K 10 |  |  | －5 | 28－150 M |  | ${ }_{24}{ }^{74}-133 \mathrm{M}$ |  | －$\times 17$ |  |  |  |  |
| $=6$ |  |  | $1 \times 12$ | － 1 | $\begin{array}{ll}169 & 211 \\ 512\end{array}$ |  | 5 k 15 | $-\frac{1}{2}$ | 64 11 |  | $0 \times 21$ |  |  |
| 0 3 |  | 1 | ${ }_{81}^{336}$－${ }_{146}^{305}$ |  |  |  |  |  |  | 0 | 376－428 |  |  |
|  |  | 7 | $81-146 \%$ 60 |  | $9 \times 13$ | －1 | $96-1172$ 296 |  | $5 \times 17$ | 3 | 371－361 |  |  |
|  | 8 K 10 |  | 60 166 | － 7 | 26.137 M | 2 | 194－209 | 3 | 785832 | 8 | 36152 |  |  |
| － 5 | 36144 M |  | 2 K 12 | $\stackrel{1}{4}$ |  | 3 | 84107 | 0 | 394－399 |  | 1 k 21 |  |  |
| － 2 | $1000{ }^{1086}$ | 1 |  | 2 | $44 \quad 131$ |  | 6 K 15 |  | $6 \times 17$ | 1 |  |  |  |
| 1 | 239．－ 38.157 | 2 | 75－124 M |  |  |  | K |  | $185 \mathrm{Cl4}$ | 4 | $537-533$ |  |  |
|  |  | 8 | 334  <br> 353 304 <br> 989  |  | $10 \times 13$ | －3 | $\begin{array}{cc}172 \\ 44 & 137 \\ 141\end{array}$ | 1 | $185 \quad 274$ |  |  |  |  |
|  | $9 \times 10$ |  |  | 9 | 83－102M | 3 | 15－123 M |  | $7 \times 17$ |  |  |  |  |
| 7 |  |  | 3 K 12 | $=6$ | ${ }_{30}^{390}{ }_{80}{ }^{426}$ M |  |  |  |  | 1 | 103－152 M |  |  |
| 4 |  |  | 80.116 M | $-{ }_{0}$ |  |  | 7 K 15 | 2 | 173188 | 5 | $\begin{aligned} & 30 \\ & 283-203 \end{aligned}$ |  |  |
| －1 | 271 <br> 572 |  | $276=282{ }^{26}$ |  |  |  | 348－321 |  | K 17 |  |  |  |  |
| 2 | $572 \quad 538$ | 6 | 35－164 M |  | $11 \times 13$ | － 2 |  |  | K 17 |  | K 21 |  |  |
|  | $10 \times 10$ |  | 4 K 12 |  |  | 2 | 137130 M | － | 11t－103m | 0 | 40－139 M |  |  |
|  |  |  |  | － 5 | 122－102 M |  | ¢ K 15 |  |  | 0 | $40-239 \mathrm{~N}$ |  |  |
| －9 |  | －${ }_{1}^{2}$ | ${ }_{100}^{133}-124 \mathrm{M}$ | －2 | 178153 |  |  |  | 0 K 18 |  | 4 K 21 |  |  |
| － 3 | ${ }_{82}^{152}=220 \mathrm{~m}$ | 4 | 288 ${ }^{280}$－ 292 |  |  | $=4$ | ${ }_{69}^{69}$－ 137 M |  |  | 1 | 66－121 M |  |  |
|  | 82－． 205 M | 7 | $400-471$ |  | $0 \times 14$ | － 1 | 267－ 262 | 3 | 226 656 $=239$ |  |  |  |  |
|  | 11 K 10 |  | $5 \times 12$ |  | 228－202 |  | 196－190 |  |  |  | $5 \times 21$ |  |  |
|  | 286226 |  |  | 4 | ${ }_{324}^{118}=128 \mathrm{~m}$ |  | 9 K 13 |  | 1 k 16 | 2 | 150－137 |  |  |
| － 5 | 116－119 M | －1 | 1975－226 | 7 |  |  |  |  |  |  | 6 k 21 |  |  |
| － 2 | 175－191 | ${ }_{5}^{2}$ |  |  | $1 \times 14$ | － $\begin{array}{r}0 \\ -0\end{array}$ | 1998939 | 4 | $\begin{aligned} & 19-326 \mathrm{M} \\ & 17 \\ & 100 \end{aligned}$ | 0 |  |  |  |
|  | 12 K 10 |  |  |  | 9－103 M |  |  |  |  | － | 210－307 |  |  |
| － 7 | 170186 |  | 6 K 12 | ${ }_{8}$ |  |  | $10 \times 15$ |  | $2 \times 18$ |  |  |  |  |
| －4 | $9 \quad 79 \mathrm{M}$ | －3 |  |  |  | － |  |  |  |  | $0 \times 22$ |  |  |
|  |  | 0 | 263 306 |  | 2 K 14 | －3 | $60.105 \%$ | 2 | 75 $=166 \mathrm{~m}$ | 2 | 182245 |  |  |
|  | $0 \times 11$ | ${ }_{6}$ | ${ }_{96}^{306}$－${ }^{392} \mathrm{El}$ |  | $134 \quad 134$ | －2 | 5186 M | 5 | 546578 | 5 | $110 \quad 84 \mathrm{M}$ |  |  |
|  |  |  |  | 3 | 63 133 m |  |  |  | 3 K 18 |  | 1 K 22 |  |  |
| 1 | ${ }_{406}^{99} \cdot 317$ |  | $7 \times 12$ | 6 | 220－284 |  | $0 \times 16$ |  |  |  |  |  |  |
| ${ }^{7}$ | $141.153 M$ |  | 96－166 M |  | $3 \times 14$ |  |  | 0 | 46170 m | 0 | 23－193M |  |  |
| 10 | 5－103 n | － 2 | ${ }^{46}$ 46 $166 ⿳ ⺈ 冂$ |  | 12321220 | 5 | ${ }^{170} 189$ |  | K ${ }^{18}$ | 3 |  |  |  |
|  | 1 K 11 | 4 | 159  <br> 523 $166 \%$ <br> 41  | － 2 | 1232 523 | 8 | 117179 |  | 272 |  | $2 \times 22$ |  |  |
|  |  |  | $8 \times 12$ | 4 | ${ }_{221}^{116}$－${ }_{234}^{150}$ M |  | $1 \times 16$ | 1 | 23 166 M | 1 | $940-1121$ |  |  |
| 5 | ${ }^{165}$－ 142 m m |  | $8 \times 12$ |  | 221534 |  |  |  | 3 K 18 | 4 | 309－213 |  |  |
| 8 | 278－320 | －7 | 523 141 2 |  | －K 14 | ${ }_{6}^{3}$ | 269 80 80 |  |  |  | $3 \times 22$ |  |  |
|  | 2 k 11 | －1 | 12.166 M |  | 373－361 |  |  | 2 | $86-133 \mathrm{M}$ | 2 | 94.163 |  |  |
|  |  | 2 | $10 \quad 133 \mathrm{~m}$ | 2 | 11461214 |  | 2 K 16 |  | $6 \times 18$ |  |  |  |  |
|  |  |  | 9 K 12 | 5 | 231257 |  |  |  |  |  | 4 K 22 |  |  |
| ${ }_{6}^{6}$ | 12 150 <br> 100  <br> 18  |  | 9 K 12 |  | 5 K 14 | 4 | 243－ $263{ }^{69}$ M | － |  | 0 | 39107 M |  |  |
| 9 | 10096 M | $=6$ | ${ }^{6} 164{ }^{\text {M }}$ |  |  | 7 | 339545 |  | $7 \times 18$ |  |  |  |  |
|  | 3 K 11 | 0 |  | － 0 | ${ }_{14}^{44^{-1} 143 \mathrm{~m}}$ |  |  | 1 |  |  |  |  |  |
|  |  |  |  |  | 132－146 M |  |  |  |  |  | 2 |  |  |
|  | 110.112 m |  | 10 K 12 |  | 107 －8 | －${ }_{2}$ | $\begin{aligned} & 12-103 \mathrm{M} \\ & 92-130 \mathrm{M} \end{aligned}$ |  | －K 19 | 1 | $\begin{array}{r} 61 \\ 103 \end{array}-\quad \begin{gathered} 217 \\ \hline 77 \end{gathered}$ |  |  |



Fig. 3. $R$ phase structure: atomic coordination. 6-coordinated ligands shown as double lines. For clarity many atoms (equivalent to other atoms shown) are omitted.
which yields a primitive cell. The basal plane of this cell is parallel to ( $\overline{1} \overline{3} 5)_{\text {hex. }}$ (equivalent to $(4 \overline{1} 5)_{\text {nex. }}$ ), very nearly the most strongly reflecting plane in the structure. The second order reflection, $(\overline{2}, \overline{6}, 10)$ (or $(8, \overline{2}, 10))$, is also very strong. On this plane the cell edges are:

$$
a_{0}^{\prime}=9 \cdot 01, \quad b_{0}^{\prime}=34 \cdot 01, \quad \gamma^{\prime}=92^{\circ} 59^{\prime}
$$

For comparison the corresponding cell edges in the orthorhombic $P$ phase and tetragonal $\sigma$ phase are:

$$
\begin{array}{cr}
P & \sigma(\mathrm{Fe}-\mathrm{Cr}) \\
a_{0}=9.07 \AA & a_{0}=8.80 \AA \\
b_{0}=16.98 &
\end{array}
$$

However, the $\mathbf{c}^{\prime}$ axis in the transformed cell is not even approximately orthogonal to the other two:

$$
c_{0}^{\prime}=9 \cdot 01 \AA, \quad \alpha^{\prime}=23^{\circ} 23 \cdot 6^{\prime}, \quad \beta^{\prime}=74^{\circ} 27 \cdot 5^{\prime}
$$

The spacing of the 'main layers' (the interplanar spacing of $(\overline{1} \overline{3} 5))$ is $2 \cdot 17 \AA$, while in the $P$ and $\sigma$ phases it is one half the $c$-repeat, or $2 \cdot 38$ and $2 \cdot 27 \AA$ respectively.

The arrangement of the atoms lying on or close to
the plane at $z^{\prime}=\frac{1}{2}$ is roughly describable as a superstructure on a main layer of the $P$ phase or the $\sigma$ phase, as may be seen in Fig. 4. Much the same arrangement of hexagonal and pentagonal 'holes' is found in the greater part of the main layer ('kagomé tiling'; Frank \& Kasper, 1959) as is found in the $P$ phase. Over most of the cell the deviations of the main-layer atoms from $z=\frac{1}{2}$ are generally less than about $0 \cdot 1$ parameter unit, or $0.2 \AA$. There is, however, a region in the cell (where ligands are drawn with dashed lines in Fig. 4) which represents a transition from one unit cell to the next and the termination of linear rows of hexagonal and pentagonal 'holes'. In this transition region there are atoms between the 'main' and 'subsidiary' layers. Owing to the obliqueness of the $\mathbf{c}^{\prime}$ axis, these transition regions lie sandwiched between more-or-less normal main-layer regions. The higher-coordinated atoms, with their larger radii, tend to be concentrated in and around these transition regions.

The $R$ phase structure resembles the $\mu$ phase structure (Arnfelt \& Westgren, 1935) in being rhombohedral, though lower in symmetry and much more complicated than that structure ( $D_{3 d}^{5}-R \overline{3} m, 13$ atoms per rhombohedral unit cell). The $\mu$ phase and the Laves phases, like the $R$ phase, possess strings of fused polyhedra along three-fold axes (see Fig. 5). Probably


Fig. 4. $R$ phase structure: normal projection on ( $\overline{13} 5$ ), showing resemblance to $P$ and $\sigma$ phase structures. Dashed lines denote ligands to atoms not lying either in the main or in the subsidiary layers.
the most significant relationship of the $R$ phase with the $\mu$ phase, as also with $\sigma$ and $P$ and other related phases, lies in the occurrence of coordination polyhedra with coordination numbers $12,14,15$, and 16 (designated in the figures and tables as the respective types, $A, B, C, D$ ) and in the manner of their combination so as to yield packing that is essentially tetrahedral throughout (i.e., tetrahedral interstices only). These polyhedra have been discussed by Kasper (1956) in relation to complex alloy structures generally. They have been discussed in connection with the $P$ and $\sigma$ phase structures by Shoemaker, Shoemaker \& Wilson (1957) and in connection with a number of structures including $P$ and $\sigma$ by Frank \& Kasper (1958, 1959). As in the case of the other structures, the ligands in the $R$ phase can be divided into two types, 5 -coordinated and 6 -coordinated; the latter being generally shorter, due in part perhaps to packing geometry but


Fig. 5. $R$ phase and some related structures: Strings of fused polyhedra along 3 -fold axes. (a) $R$ phase, entire c-repeat: CN 16, 12, $12,12,16$. (b) $\mu$ phase, one-half $c$-repeat: CN 12, $15,16,14$. (CN 12 is shared with lower half, below.) (c) $\mathrm{MgNi}_{2}$, entire c-repeat: $\mathrm{CN} 16,12,12,16$ ).
also, presumably, in part to stronger bonding. These are the types referred to by Frank \& Kasper as 'minor' and 'major' ligands respectively.
The 6 -coordinated ('major') ligands form a single three-dimensional network ('major skeleton') with branching at the 15 - and 16 -coordinated atoms, as shown in Fig. 6. In the fact that all atoms forming major ligands (coordinations $14,15,16$ ) belong to a single all-pervading network, the $R$ phase resembles the simple Laves phases ( $\mathrm{MgCu}_{2}, \mathrm{MgZn}_{2}$ or $\mathrm{FeW}_{2}$, $\mathrm{MgNi}_{2}$ ), and differs from the more complex $\mu, \sigma$ and $P$ phases in which unconnected networks exist with layering and/or interpenetration. The structure of the $R$ phase network is, however, very complicated, and simple or meaningful relationships with those of the other structures are hard to find.


Fig. 6. $R$ phase structure: Network of 6-coordinated ligands. Gradations in heaviness of lines increases with the hexagonal $z$ coordinate from 0.00 to 1.00 .

In this work, as in the work on the $P$ phase, an attempt was made to deduce, from the interatomic distances, 5 -coordinated and 6 -coordinated radii for each kind of atom, by least-squares. The resulting radii, and their averages for each coordination type, are given in Table 5. 'Predicted' interatomic distances, calculated with these average radii, are given in Table 6, for comparison with the observed interatomic
distances, given in Table 4. The differences are somewhat larger than for the $P$ and $\sigma$ phases, averaging to $0.056 \AA$. in comparison to $0.036 \AA$ for $P$ and $0.039 \AA$ for $\sigma$. The poorer agreement may be attributed in some part to lower precision in the interatomic dis-
tances, owing to the relatively scanty intensity data; however, it is significant that there are eight differences in excess of $0 \cdot 1 \AA$, the largest being $0 \cdot 174 \AA$, by contrast with only one, $0 \cdot 122 \AA$, in the $P$ phase. Moreover, this seems to be an inherent property of

Table 4. Observed interatomic distances


Table 4 (cont.)


| Atom | No. | R | $p q r$ | $D_{0}$ | $\Delta$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | D2 |  |  |
| A3 |  |  |  | $2 \cdot 679$ | 103 |
| A3' |  | $\overline{3}$ |  | 2.765 | 17 |
| A4 |  | $3^{2}$ |  | 2.818 | -36 |
| A4' |  | $\overline{3}^{5}$ | 211 | $2 \cdot 817$ | -35 |
| A4" |  |  |  | 2.757 | 25 |
| A5 |  | $3^{2}$ |  | 2.721 | 61 |
| $A 5^{\prime}$ |  | $\overline{3}^{5}$ |  | $2 \cdot 674$ | 108 |
| $A 5^{\prime \prime}$ |  |  |  | 2.763 | 19 |
| $A 6$ |  |  |  | 2.857 | -75 |
| $A 6^{\prime}$ |  | $\overline{1}$ | 211 | 2.738 | 44 |
| B1 |  | $3^{2}$ | 121̄ | 3.063 | -41 |
| B2 |  | $3^{2}$ | 121 | 3.196 | -174 |
| * $\mathbf{B 2}$ |  | $\overline{1}$ | 122 | $2 \cdot 700$ | -52 |
| ${ }^{*} \mathrm{Cl}$ |  | $\overline{3}^{5}$ | 211 | $2 \cdot 722$ | 21 |
| *D1 |  |  |  | $2 \cdot 859$ | 13 |
| * ${ }^{2} 2$ |  | $\overline{1}$ | 211 | $2 \cdot 868$ | 4 |
|  |  |  |  |  | 52 |
|  |  | Over | weig | d aver | 56 |

Atom: Designation of near neighbor. Primes distinguish among non-equivalently disposed neighbors of the same crystallographic kind.
No.: Number of equivalently disposed neighbors of that kind, if different from unity.
$R, p q r: \quad$ Operation by which the neighbor is obtained from the one with coordinates listed in Table 2. $R$ is the rotation or rotatory inversion or inversion, and $p q r$ represents a subsequent translation $p / \mathbf{3}, q / 3, r / 3$.
$D_{o}$ : Interatomic distances in $\AA$ calculated from parameters given in Table 2.
$\Delta$ : Amount in units of $0.001 \AA$, by which the predicted distance (Table 6) differs from $D_{o}$.
the structure, inasmuch as attempts to 'refine' the parameters without intensity data gave substantially the same large differences.

Table 5. Summary of radii

| Atom | No. | $r$ | $r^{*}$ |
| :---: | :---: | :---: | :---: |
| $A 1$ | 1 | $1.208 \AA$ |  |
| $A 2$ | 2 | 1.198 |  |
| $A 3$ | 6 | 1.174 |  |
| $A 4$ | 6 | 1.180 |  |
| $A 5$ | 6 | 1.198 |  |
| $A 6$ | 6 | 1.176 |  |
| $A$ | 27 | 1.184 |  |
| $B 1$ | 6 | 1.422 | $1.210 \AA$ |
| $B 2$ | 6 | 1.426 | 1.215 |
| $B$ | 12 | 1.424 | 1.212 |
| $C$ | 6 | 1.498 | 1.307 |
| $D 1$ | 2 | 1.603 | 1.419 |
| $D 2$ | 6 | 1.597 | 1.442 |
| $D$ | 8 | 1.598 | 1.436 |
| Total | 53 |  |  |
|  |  |  |  |

Nevertheless the agreement is good enough to corroborate the indication obtained with the $\sigma$ and $P$ phase results that the ligand radii should conform
in considerable degree to the symmetry of the idealized coordination polyhedron, and should be essentially constant for a given atom coordination and given ligand coordination. As with the $P$ phase the coordination type is in part determined by atomic size, as shown by the indications that the more highly coordinated sites are occupied predominantly by molybdenum atoms.

Table 6. Predicted interatomic distances

| $A-A$ | $2.368 \AA$ | ${ }^{*} B-B$ | $2.424 \AA$ |
| :--- | :--- | :--- | :--- |
| $A-B$ | 2.608 | $*_{B}-C$ | 2.519 |
| $A-C$ | 2.682 | $* B-D$ | 2.648 |
| $A-D$ | 2.782 |  |  |
|  |  |  |  |
| $B-B$ | 2.848 | $* C-C$ | 2.614 |
| $B-C$ | 2.922 | $* C-D$ | 2.743 |
| $B-D$ | 3.022 |  |  |
|  |  | $* D-D$ | 2.872 |

The results of this and previous investigations give some ground for new speculations as to the filling of atomic orbitals or energy bands in structures of these types. It is possibly significant that the degeneracy of the $d$ orbitals is not removed under the symmetry group ( $Y_{h}$ ) of the regular icosahedron. This fact suggests that in the coordination 12 atoms the $d$ band prefers to be substantially filled (or perhaps in some
cases substantially empty). These sites tend to be occupied by elements at and to the right of Mn , and the atoms would be somewhat negative by electron transfer. In the other coordinations the $d$ degeneracy is removed, and $d, s$, and $p$ orbitals might be expected to hybridize to form directed bonds in the 6 -coordinated ligand directions. These sites tend to be occupied by elements at and to the left of Cr and Mo (Kasper \& Waterstrat, 1956), with Mo replacing Cr in the higher coordinations because of larger size; these atoms would then be somewhat positive by electron transfer. These considerations will be discussed more in detail elsewhere (Shoemaker, 1960).

Some preliminary single-crystal exploration was done by Dr Clara B. Shoemaker; reference to her photographs materially aided in the establishment of the monocrystallinity of the crystal fragment which we used. Mr Paul Metzger assisted with the refinement of the lattice constants with powder data.

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# X-Ray Diffraction Studies of the $\delta$ Phase, Mo-Ni* 

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#### Abstract

Single-crystal X-ray diffraction studies of the $\delta$ phase, Mo-Ni ( $61 \cdot 1$ weight percent Mo) have shown that it is at least pseudotetragonal, with $a_{0}=9 \cdot 108, c_{0}=8 \cdot 852 \AA$, and extinctions suggesting space group $D_{4}^{6}-P 4_{2} 2_{1} 2$. There are, however, a few cases of differing intensity between ( $h k l$ ) and ( $k h l$ ), indicating a reduction of the true Laue symmetry to $D_{2 h}$; the true space group is perhaps $P 2_{1} 2_{1} 2$ or $P 2_{1} 2_{1} 2_{1}$. The ( $0 k l$ ) and ( $h 0 l$ ) weighted reciprocal lattice nets closely resemble the ( $h k 0$ ) net of the $\sigma$ phase. All attempts to determine the structure in detail were unsuccessful.


## Introduction

In connection with work in these laboratories on the crystal structures of the $P$ phase, Mo-Ni-Cr (Shoemaker, Shoemaker \& Wilson, 1957) and the $R$ phase Mo-Co-Cr (Komura, Sly \& Shoemaker, 1960), we undertook to investigate the crystal structure of the $\delta$ phase, Mo-Ni, discovered by Ellinger (1942). This

[^2]phase appeared to be closely related to the $\sigma$ phase (Bergman \& Shoemaker, 1954) and to the $P$ phase, as was demonstrated clearly in the work of Beck and coworkers (Rideout et al., 1951) who found $\sigma, P$, and $\delta$ phase regions arranged in a roughly linear row on the $\mathrm{Mo}-\mathrm{Ni}-\mathrm{Cr}$ phase diagram at $1200{ }^{\circ} \mathrm{C}$. The $\delta$ phase in the $\mathrm{Mo}-\mathrm{Ni}$ system exists at the approximate atomic ratio $1: 1$, in the range 61 to 63 weight percent molybdenum.

## Experimental

A specimen of this alloy, annealed at $1200{ }^{\circ} \mathrm{C}$., was kindly supplied by Prof. Paul A. Beck of the Univer-


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[^1]:    * \% Mo assumed in a linear combination of Mo and Co form factors used in calculating structure factors.

[^2]:    * Sponsored by Office of Ordnance Research. Computations were done in part at the MIT Computation Center, in part on X-RAC at Pennsylvania State University.
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